

5 Time Series Analysis

5.1 Introduction

5.2 Generating Signals

5.3 Fourier Transforms FT, DFT, and FFT

Furthermore, the data $x(t)$ are only available in a limited interval, that is, between t_1 and t . The *discrete Fourier transform* (DFT)—as opposed to the continuous Fourier transform (FT)—converts such a time series $x(t)$ into a discrete Fourier (or frequency) series $X(f)$:

In some fields, the *power spectral density* (PSD) is used as an alternative method for describing the power spectrum, whereby the spectral power is divided by the sampling frequency f_s and by the length of the time series n . The *fast Fourier transform* (FFT) was introduced by Cooley and Tukey (1965).

We can use the FFT to calculate a Fourier spectrum of the signal, which is displayed as the spectral power against the frequency f and the period τ (Fig. 5.6). The function `fft` computes the Fourier transform for `nfft` frequencies, where `nfft` is the smallest power of two that is greater than or equal to the data length of the series x and is padded with zeros in order to make up the number of data points equal to the value of `nfft`.

5.4 Schuster's Periodogram Method

for the Nyquist range $-0.5 f_s < f < +0.5 f_s$, where f_s is the sampling frequency and n is the length of the time series.^s

5.5 The Blackman–Tukey Method and Welch's Method

5.6 Interpolating and Analyzing Unevenly Spaced Data

Both synthetic data sets contain a two-column array with 339 rows. The first column contains ages in *kiloyears* (*kyr*), which are unevenly spaced.

The Signal Processing Toolbox also contains the *Signal Analyzer* app, which can be started with `signalAnalyzer` in the Command Window.

5.7 The Spectrogram (Sonogram, Sonograph) Method

5.8 Thomson's Multitaper Method

As we can see, Thomson's multitaper PSD estimate is significantly smoother, which results in the greater visibility of the maximum at $f_2 \approx 0.07$. However, neither the periodogram method nor Thomson's multitaper

method is able to distinguish the high frequency peak at $f_3=0.2$ from the noise.

5.9 The Lomb–Scargle Power Spectrum

5.10 The Wavelet Power Spectrum

5.11 Nonlinear Time Series Analysis (by N. Marwan)

Considering the example of sediment in a paleolake, the sampled variable could be a time series $x(t)$ of potassium concentrations along a lake sediment core that represents a natural archive of past states of the lake system.

One way to unfold the dynamics of a multi-dimensional system from a one-dimensional time series $x(t)$ is through time delay *embedding*, which preserves the dynamic characteristics of the system (Packard et al. 1980).

The embedding of the time series $x(t)$ in a three-dimensional ($m=3$) *phase space*, for example, means that three successive values $x(t)$, $x(t+\tau)$, and $x(t+2\tau)$ with a temporal separation τ are represented as a point in a three-dimensional coordinate system (i.e., the phase space) (Packard et al. 1980). The *phase space portrait* displays the embedded time series of observations as a trajectory $y(t)=[x(t), x(t+\tau), x(t+2\tau)]^T$ in the phase space.

5.12 Detecting Abrupt Transitions in Time Series

5.13 Describing Gradual Transitions in Time Series

The synthetic data sets are then analyzed using the function `fit` together with `fitoptions` and `fitttype`, which are included in the **Curve Fitting Toolbox** of MATLAB for performing nonlinear least squares fitting (MathWorks 2025d).

5.14 Aligning Stratigraphic Sequences

Recommended Reading

Figure Captions